
Course **Financial Mathematics**

Unit course **FIN 118**

Number Unit **12**

Unit Subject **Bond Valuation**

Dr. Lotfi Ben Jedidia
Dr. Imed Medhioub



we will see in this unit

- ✓ What's a bond
- ✓ Different types of bonds
- ✓ Bond valuation



LEARNING OUTCOMES

At the end of this chapter, you should be able to:

1. Understand what is meant by "Bond".
2. Know the different types of bonds.
3. Calculate the value of a bond.



What is a Bond

Definition:

- A debt instrument: When one purchases a bond, one essentially lends an organization such as the government or a corporation a specified amount of money which the borrower agrees to repay at a designated time.
- A promise to pay interest over a specific term at stated future dates and then pay lump sum at maturity (the end of the term).
- Issued by corporations and governments as a way to provide money to the company.



Components of a bond

- Principal or Face value of the bond: The amount of money that is paid to the bondholders at maturity. For most bonds this amount is \$1,000 (and its doubles). It also generally represents the amount of money borrowed by the bond issuer.
- Coupon rate or rate of interest : It is expressed as a percentage of the bond's face value. It also represents the interest cost of the bond to the issuer.



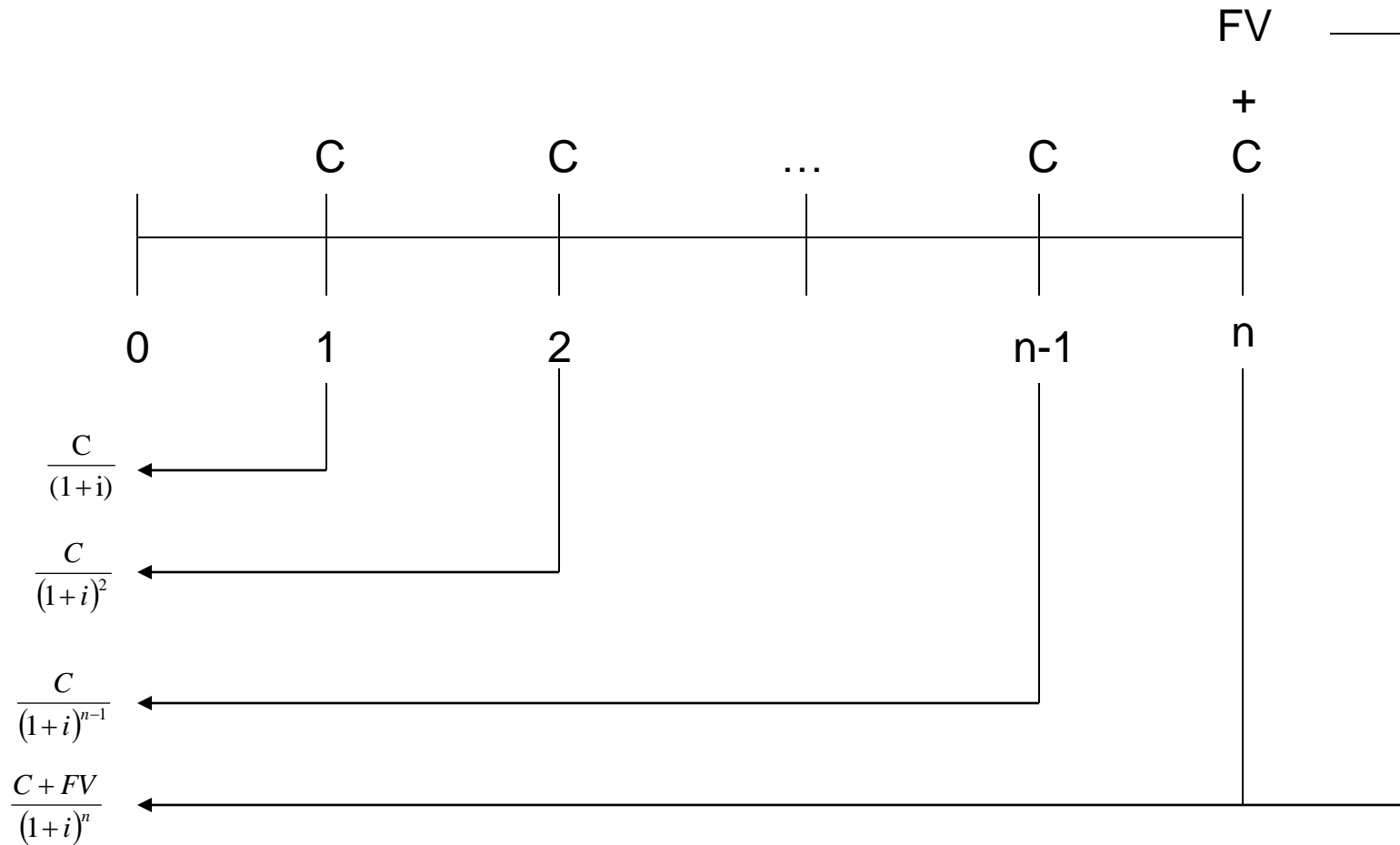
Components of a bond

- **Coupon Payments:** It represent the periodic interest payments from the bond issuer to the bondholder. The annual coupon payment is calculated by multiplying the coupon rate by the bond's face value. Since most bonds pay interest semiannually, generally one half of the annual coupon is paid to the bondholders every six months.
- **Maturity date:** It represents the date on which the bond matures, i.e., the date on which the face value is repaid. The last coupon payment is also paid on the maturity date.



Bond Valuation

Time line of payments



Types of Bonds

- **Government Bonds:** or Treasury Bonds, a debt security issued by a government to support government spending.
- **Corporate Bonds:** a debt security issued by companies and sold to investors in order to finance expansion or raise funds for other expenses. Interest rates accorded to this type of bonds is higher than government bonds.
- **Municipal Bonds:** a debt security issued by a state, a municipality or a county in order to finance its projects or expenditures. Municipal bonds may be general obligations of the issuer or secured by specified revenues.



Bond Valuation

- Bonds are valued using time value of money concepts.
- Their coupon, or interest, payments are treated like an equal cash flow stream (annuity).
- Their face value is treated like a lump sum.

$$PV(\text{Bond}) = \sum_{t=1}^n \frac{\text{Coupon}}{(1+i)^t} + \frac{\text{Face Value of Bond}}{(1+i)^n}$$

$$PV(\text{bond}) = \text{Coupon} \times \left[\frac{1 - (1+i)^{-n}}{i} \right] + \frac{\text{Face Value of bond}}{(1+i)^n}$$

$$\text{Coupon} = \text{Face Value} \times \text{Coupon rate}$$

C = Coupon; r = annual coupon rate; i = annual interest rate;
FV = face value; n = number of years.



Bond Valuation

Example 1: Annual Coupon Payments

Your father bought a 10-year bond from Al-Nasr Corporation Europe Ltd. The bond has a face value of \$1000 and pays an annual coupon rate 10%. The current market rate of return is 12%.

- 1- Find the annual value of the coupon.
- 2- Find the Price of the Bond (Market value of the bond to day).



Bond Valuation

Solution:

1/ Annual value of the coupon

$$\begin{aligned}\text{Coupon} &= \text{Face Value} \times \text{coupon rate} \\ &= 1000 \times 10\% = \$100\end{aligned}$$

2/ The bond price is the Present value of the coupon stream plus the Present Value of the Face Value.

$$\begin{aligned}\text{Bond Price} &= 100 \times \left[\frac{1 - (1 + 0.12)^{-10}}{0.12} \right] + \frac{1000}{(1 + 0.12)^{10}} \\ &= \$565.02 + \$321.97 = \$886.99\end{aligned}$$



Bond Valuation

Non-annual Coupon Payments

- The rule for valuing annual bonds is easily extended to valuing bonds paying interest even more frequently (semi-annually, quarterly, monthly).
- For example, to determine the value of a bond paying interest semi-annually, we can remark that we have to pay the coupon two times a year. Then, to calculate the price of the bond, we must double the number of annual periods and the annual coupon payment and divide the coupon rate and the discount rate by two.



Bond Valuation

Non-annual Coupon Payments

- In general, if we let t be equal to the number of payments per year, n be equal to the maturity in years and i be the annual discount rate, then the general formula for valuing a bond can be expressed as follows:

$$PV(\text{bond}) = \text{Coupon} \times \left[\frac{1 - \left(1 + \frac{i}{t}\right)^{-n \times t}}{\frac{i}{t}} \right] + \frac{\text{Face Value of bond}}{\left(1 + \frac{i}{t}\right)^{n \times t}}$$

$$\text{Coupon} = \text{Face Value} \times \frac{\text{Coupon rate}}{t}$$

C = Coupon; r = coupon rate; i = interest rate; FV = face value; n = number of years; t = number of times in 1 year.



Bond Valuation

Example 2: Semi-Annual Coupon Payments

Your father bought a 10-year bond from Al-Nasr Corporation Europe Ltd. The bond has a face value of \$1000 and pays a semi-annual coupon rate 10%. The current market rate of return is 12%.

- 1- Find value of the coupon.
- 2- Find the Bond price.



Bond Valuation

Solution:

1/ Annual value of the coupon

$$\begin{aligned}\text{Coupon} &= \text{Face Value} \times \text{coupon rate}/2 \\ &= 1000 \times (0.1/2) = \$50\end{aligned}$$

2/ The bond price is the Present value of the coupon stream plus the Present Value of the Face Value.

Then,

$$\begin{aligned}\text{Bond Price} &= 50 \times \left[\frac{1 - \left(1 + \frac{0.12}{2}\right)^{-10 \times 2}}{\frac{0.12}{2}} \right] + \frac{1000}{\left(1 + \frac{0.12}{2}\right)^{10 \times 2}} \\ &= \$573.496 + \$311.805 = \$885.30\end{aligned}$$



Relation between coupon rate and discount rate

First Relation:

three cases are possible:

✓ Coupon rate = discount rate

The price of the bond equal to the Face value of the bond \Rightarrow **par bond**

✓ Coupon rate > discount rate

The price of the bond is greater than the Face value of the bond \Rightarrow **premium bond**

✓ Coupon rate < discount rate

The price of the bond is smaller than the Face value of the bond \Rightarrow **discount bond**



Relation between bond price and discount rate

Second Relation:

Two cases are possible:

✓ Rate of return increase \Rightarrow The price of the bond decreases

✓ Rate of return decrease \Rightarrow The price of the bond increases

✓ Inverse relation between bond price and discount rate

$$\frac{\Delta BP}{\Delta i} < 0$$



Bond Valuation

Example 3:

Your father bought a 10-year bond from Al-Nasr Corporation Europe Ltd. The bond has a face value of \$1000 and pays an annual coupon rate 10%.

- 1- Find the price of the bond if the current market rate of return is 8%. What we can conclude?
- 2- Find the price of the bond if the current market rate of return is 10%. What we can conclude?
- 3- Find the price of the bond if the current market rate of return is 12%. What we can conclude?



Bond Valuation

Solution:

$$\text{Coupon} = 0.1 \times 1000 = \$100$$

1- If the market interest rate is 8% (the discount rate), the market value of the bond is:

$$\text{Bond Price} = 100 \times \left[\frac{1 - (1 + 0.08)^{-10}}{0.08} \right] + 1000 \times (1 + 0.08)^{-10} = \$1134.201$$

We have a **premium bond**.

2- If the market interest rate is 10% (the discount rate), the market value of the bond is:

$$\text{Bond Price} = 100 \times \left[\frac{1 - (1 + 0.1)^{-10}}{0.1} \right] + 1000 \times (1 + 0.1)^{-10} = \$1000$$

We have a **par bond**.



Bond Valuation

Solution:

3- If the market interest rate is 12% (the discount rate), the market value of the bond is:

$$\text{Bond Price} = 100 \times \left[\frac{1 - (1 + 0.12)^{-10}}{0.12} \right] + 1000 \times (1 + 0.12)^{-10} = \$886.995$$

We have a **discount bond**.

What happens if coupon payments are treated semi-annually?



Time to Review !

- ✓ Bonds are debt instruments with maturity date.
- ✓ If the coupon rate is greater than the market rate, the market value of the bond is greater than the Face value of the bond.
- ✓ If the coupon rate is smaller than the market rate, the market value of the bond is smaller than the Face value of the bond.
- ✓ If the two rates are equal, market value is equal to the Face value of the bond.



we will see in the next unit

- ✓ Meant of Equal short term payments and settlement of short-term debt
- ✓ How to Calculate the amount of total payments.
- ✓ How to Calculate the amount of a new settlement

