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Course	**Financial Mathematics**
Unit course	**FIN 118**
Number Unit	**11**
Unit Subject	**Long Term Annuities Amortization & sinking Funds**

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we will see in this unit

- ✓ Long term ordinary annuity
- ✓ Long term annuity due
- ✓ Amortization & sinking Funds
- ✓ Some real life examples

LEARNING OUTCOMES

At the end of this chapter, you should be able to:

1. Understand what is meant by "long term annuities".

2. Calculate Present and future value long term annuities for the case of ordinary and annuity due.

3. Calculate the single payment for real life examples in the case of long term annuities (Amortization & sinking Funds).

Introduction

- As we say in unit 10, Annuity is a series of equal cash payments or deposits. These regular equal payments can be planned for short term, intermediate or long term periods.

- In this unit we are interested to the calculation of long term payments or deposits.

- Two basic questions can be exposed with annuities:
  - Calculate how much money will be accumulated if we consider an annuity plan for long time (30 years for example)
  - How to calculate the periodic payments or deposits in order to obtain a specific amount on a given time period (calculate monthly payments for a mortgage loan for example).

Long Term Ordinary Annuity

- As we have seen in unit 10, the same formula is applied to the case of long term annuities.
- The future and present value of an ordinary annuity are given respectively by

$$FVA_n = PMT\left[\frac{(1+i)^n - 1}{i}\right] \qquad PVA_n = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

if payments are made annually.

- If payments are made non annually (more than once in year) we must introduce in the previous formula the number of times per year. So, we have the following formula

$$FVA_{n,t} = PMT\left[\frac{\left(1+\frac{i}{t}\right)^{n.t} - 1}{\frac{i}{t}}\right] \qquad PVA_{n,t} = PMT\left[\frac{1-\left(1+\frac{i}{t}\right)^{-n.t}}{\frac{i}{t}}\right]$$

Long Term Ordinary Annuity

Example1

You plan to deposit in an account \$5000 at the end of each year for the next 25 years. If the account pays 6% annually, what is the value of your deposits at the end of 25 years?

Solution

$$FVA_{25} = 5000\left[\frac{(1+0.06)^{25}-1}{0.06}\right] = \$274322.56$$

Long term Ordinary Annuity

Example2

You plan to deposit, in an account, $100 at the end of each month for the next 25 years. If the account pays 6% annually, what is the value of your deposits at the end of 25 years?

Solution

$$FVA_{25,12} = 100\left[\frac{\left(1+\frac{0.06}{12}\right)^{(25)(12)} - 1}{\frac{0.06}{12}}\right] = \$69299.396 \approx \$69300$$

Long Term Ordinary Annuity

Example3

You plan to withdraw at the end of each year from an account $5000 for the next 25 years. If the account pays 6% annually, what is the amount that you must deposit today in order to guarantee these withdrawals?

Solution

$$PVA_{25} = 5000\left[\frac{1-(1+0.06)^{-25}}{0.06}\right] = \$63916.78$$

Long term Ordinary Annuity

Example4

You plan to withdraw at the end of each month $100 from an account of the next 25 years. If the account pays 6% annually, what is the amount that you must deposit today in order to guarantee these withdrawals?

Solution

$$PVA_{25,12} = 100\left[\frac{1-\left(1+\frac{0.06}{12}\right)^{-(25)(12)}}{\frac{0.06}{12}}\right] = \$15520.69$$

Long Term Annuity Due

- As we have seen in unit 10, the same formula is applied to the case of long term annuities.
- The future and present value of an annuity due are given respectively by

$$PVA_n = PMT\left[\frac{1-(1+i)^{-n}}{i}\right](1+i)$$

$$FVA_n = PMT\left[\frac{(1+i)^n - 1}{i}\right](1+i)$$

if payments are made annually.

- If payments are made non annually (more than once in year) we must introduce in the previous formula the number of time per year. So, we have the following formula

$$FVA_{n,t} = PMT\left[\frac{\left(1+\frac{i}{t}\right)^{n.t} - 1}{\frac{i}{t}}\right]\left(1+\frac{i}{t}\right)$$

$$PVA_{n,t} = PMT\left[\frac{1-\left(1+\frac{i}{t}\right)^{-n.t}}{\frac{i}{t}}\right]\left(1+\frac{i}{t}\right)$$

Long Term Annuity Due

Example1

You plan to deposit in an account $5000 at the beginning of each year for the next 25 years. If the account pays 6% annually, what is the value of your deposits at the end of 25 years?

Solution

$$FVA_{25} = 5000\left[\frac{(1+0.06)^{25}-1}{0.06}\right](1+0.06)$$

$$= \$290781.91$$

Long term Annuity Due

Example2

You plan to deposit in an account $100 at the beginning of each month for the next 25 years. If the account pays 6% annually, what is the value of your deposits at the end of 25 years?

Solution

$$FVA_{25,12} = 100\left[\frac{\left(1+\frac{0.06}{12}\right)^{(25)(12)}-1}{\frac{0.06}{12}}\right]\left(1+\frac{0.06}{12}\right)$$

$$= \$69645.89$$

Long Term Annuity Due

Example3

You plan to withdraw from an account $5000 at the beginning of each year for the next 25 years. If the account pays 6% annually, what is the amount that you must deposit today in order to guarantee these withdrawals?

Solution

$$PVA_{25} = 5000\left[\frac{1-(1+0.06)^{-25}}{0.06}\right](1+0.06)$$

$$= \$67751.79$$

Long term Annuity Due

Example4

You plan to withdraw from an account $100 at the beginning of each month for the next 25 years. If the account pays 6% annually, what is the amount that you must deposit today in order to guarantee these withdrawals?

Solution

$$PVA_{25,12} = 100\left[\frac{1-\left(1+\frac{0.06}{12}\right)^{-(25)(12)}}{\frac{0.06}{12}}\right]\left(1+\frac{0.06}{12}\right)$$

$$= \$15598.29$$

Amortization &Sinking Funds
إطفاء/اهلاك القروض أو استهلاك القروض
صندوق إطفاء الدين أو اِحْتِيَاطِيّ سداد قرض

- When a lender pays a debt (including interest) by making periodic payments at regular intervals, the debt is said to be *amortized*.

- When a payment is made to an investment fund each period at a fixed interest rate to yield a predetermined future value, the payment is called a sinking fund.

Amortization &Sinking Funds

- Ordinary Amortization Formula

Annual	Non-annual
$PMT = PVA\left(\frac{i}{1-(1+i)^{-n}}\right)$	$PMT = PVA\left(\frac{\frac{i}{t}}{1-\left(1+\frac{i}{t}\right)^{-n\times t}}\right)$

- Ordinary Sinking Fund Payment

Annual	Non-annual
$PMT = FVA\left[\frac{i}{(1+i)^{n}-1}\right]$	$PMT = FVA\left[\frac{\frac{i}{t}}{\left(1+\frac{i}{t}\right)^{n\times t}-1}\right]$

Real life example: Loan Amortization

Example

Suppose you want to borrow money to buy a house. You are considering a 7-year or a 25-year loan.

A bank offers different interest rates, reflecting the differences in risks of intermediate-term and long-term lending.

For the 7-year loan, the annual interest rate is 5.25% compounded monthly.

For the 25-year loan, the annual interest rate is 6.75% compounded monthly. If you borrow $150000, what would be your monthly payments for each type of loan?

Real life example: Loan Amortization

Solution

First Scenario: 7-year loan

$$PMT = 150000\left[\frac{\frac{0.0525}{12}}{1-\left(1+\frac{0.0525}{12}\right)^{-(7)(12)}}\right] = \$2137.75$$

Second Scenario: 25-year loan

$$PMT = 150000\left[\frac{\frac{0.0675}{12}}{1-\left(1+\frac{0.0675}{12}\right)^{-(25)(12)}}\right] = \$1036.37$$

Real life example: Sinking Fund

Example 1

Suppose you decide to use a sinking fund to save $150 000 for a house. If you plan to make 300 monthly payments (25 years × 12=300) and you receive 6.75% interest per annum, what is the required payment for an ordinary annuity?

Solution

$$PMT = 150000\left[\frac{\frac{0.0675}{12}}{\left(1+\frac{0.0675}{12}\right)^{(25)(12)}-1}\right] = \$192.62$$

Real life example: Sinking Fund

Example 2

Suppose you use a sinking fund to save $50 000 for a car. If you plan on 60 monthly payments (5 years ×12 = 60) and you receive 5% interest per annum, what is the required payment for an ordinary annuity?

Solution

$$PMT = 50000\left[\frac{\frac{0.05}{12}}{\left(1+\frac{0.05}{12}\right)^{(5)(12)}-1}\right] = \$735.23$$

Time to Review !

- ✓Long term annuities is an extension to the ordinary and annuity due.

- ✓Making periodic payments to repay a debt, including the principal and interest, is called amortization.

- ✓A fund into which periodic payments necessary to realise a given sum of money in the future are made is called sinking fund.

we will see in the next unit

- ✓ What's a bond
- ✓ Different types of bonds
- ✓ Bond valuation