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Course	**Financial Mathematics**
Unit course	**FIN 118**
Number Unit	**1**
Unit Subject	**Linear Equations** **Quadratic Equations**

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We will see in this unit

1. Linear Equations in One Variable
   - Formulae
   - Solving a Linear Equation in One Variable
   - Applications

2. Quadratic Equations in One Variable
   - Formulae
   - Solving a quadratic Equation in One Variable
   - Applications

Learning Outcomes

At the end of this chapter, you should be able to:

1. Understand what is meant by "linear equation" and "quadratic equation"

2. Understand how to solve linear and quadratic linear equations.

3. Solve equations for real world situations in order to solve problems, especially economic and financial.

Linear equation

Definition:

Any equation written in the form

$$Ax + B = C$$

Is said a linear equation where A, B and C are fixed numbers and $A \neq 0$

Examples

- x – 5 = 16
- 2y + 4 = 12
- 5n - 4 = 6
- z/2 – 6 = 4

Linear equation

Two Steps for Solving linear Equations

Step1- Solve for any Addition or Subtraction on the variable side of equation by “undoing” the operation from both sides of the equation.

Step2- Solve any Multiplication or Division from variable side of equation by “undoing” the operation from both sides of the equation.

Linear equation

Example : Solve $4x - 5 = 15$

$$\begin{aligned} 4x - 5 &= 15 \\ \underline{+5 \quad} & \underline{\; +5} && \text{(Add 5 to both sides)} \\ \frac{4x}{4} &= \frac{20}{4} && \text{(Simplify)} \quad \text{(Divide both sides by 4)} \\ \underline{x = 5} & && \text{(Simplify)} \end{aligned}$$

- Try the above Examples:

$x - 5 = 16$; $2y + 4 = 12$; $5n - 4 = 6$; $z/2 - 6 = 4$

solutions: x = ; y = ; n = ; z =

Linear equation

Time to Review!

- Make sure your equation is in the form $Ax + B = C$
- Keep the equation balanced.
- Use opposite operations to "undo"
- Follow the rules:

    1. **Undo Addition or Subtraction**
    2. **Undo Multiplication or Division**

That's All for linear equation !

Quadratic equation

Definition:

Any equation written in the form

$$Ax^2 + Bx + C = 0$$

Is said a quadratic equation where A, B and C are fixed numbers and $A \neq 0$

Examples

$$4x^2 + x + 1 = 0$$

$$4x^2 - 4x + 1 = 0$$

$$x^2 + 8x - 20 = 0$$

Quadratic equation

How to solve quadratic equations ?

Quadratic equations can be solved using Discriminant method :

Step1 : Express the equation in a general form

$$Ax^2 + Bx + C = 0$$

Step2 : Calculate the discriminant: $\Delta = B^2 - 4AC$

Step 3: Give solution

Case1: $\Delta < 0$ no real roots

Case2: $\Delta = 0$ only one real root $r = \dfrac{-B}{2A}$

Case3: $\Delta > 0$ two distinct real roots

$$r_1 = \frac{-B - \sqrt{\Delta}}{2A} \quad \text{and} \quad r_2 = \frac{-B + \sqrt{\Delta}}{2A}$$

Quadratic equation

Example1: Solve the quadratic equation if possible

$$4x^2 + x + 1 = 0$$

Step1 : A=4, B=1 and C=1

Step2 : $\Delta = B^2 - 4AC = (1)^2 - 4(4)(1) = -15$

Step 3: $\Delta = -15 < 0$ no real roots

Quadratic equation

Example2: Solve the quadratic equation

$$4x^2 - 4x + 1 = 0$$

Step1 : A=4, B=-4 and C=1

Step2 : $\Delta = B^2 - 4AC = (-4)^2 - 4\times4\times1 = 0$

Step 3: $\Delta = 0$ only one real root

$$r = \frac{-B}{2A} = \frac{4}{2\times4} = \frac{1}{2}$$

!!! We can rewrite the quadratic equation

$$4x^2 - 4x + 1 = 4\left(x - \frac{1}{2}\right)^2 = \left(x - \frac{1}{2}\right)^2 = 0$$

Quadratic equation

Example3: Solve the quadratic equation

$$x^2+8x-20=0$$

Step1 : A=1, B=8 and C=-20

Step2 : $\Delta = B^2-4AC=(8)^2-4\times(1)\times(-20)=144$

Step 3: $\Delta=144>0$ two distinct real roots

$$r_1=\frac{-B-\sqrt{\Delta}}{2A}=\frac{-8-\sqrt{144}}{2\times 1}=\frac{-20}{2}=-10 \quad \text{and} \quad r_2=\frac{-B+\sqrt{\Delta}}{2A}=\frac{-8+\sqrt{144}}{2\times 1}=\frac{4}{2}=2$$

!!! We can rewrite the quadratic equation

$$x^2+8x-20=(x+10)(x-2)=0$$

Quadratic equation

Example: Solve the quadratic equation

$$x^2 - 4x = -3$$

Step1 : rewrite the equation in general form

A= , B= and C=

Step2 :

Step 3:

!!! We can rewrite the quadratic equation

Quadratic equation

Time to Review !

Value of $\Delta = B^2 - 4AC$	Solutions
$\Delta = B^2 - 4AC < 0$	No real solutions
$\Delta = B^2 - 4AC = 0$	One real solution
$\Delta = B^2 - 4AC > 0$	Two real solutions

- **That's All for quadratic equation !**

we will see in the next unit

1. What is meant by a function?
2. How to determine Domain and Range?
3. The properties of linear function
4. How to plot a linear function?
5. The properties of quadratic function
6. How to plot a quadratic function?

Remind these general rules

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^n \times a^p = a^{n+p}$$

$$(a^n)^p = a^{n \times p}$$

$$a^n b^n = (ab)^n$$

$$\frac{a^n}{b^m} = a^n b^{-m}$$